Critical Phenomena in Higher Dimensional Gravity Using Adaptive Mesh Refinement

University of Winnipeg, 2014

Nils Deppe
University of Winnipeg

Supervisor:
Dr. Gabor Kunstatter

May 21, 2014

Background Critical Phenomena

Gravitational collapse of massless scalar field
Initial Data (also use tanh):

$$\psi(r, t=0) = Ar^2 \exp\left[-\left(\frac{r-r_0}{B}\right)^2\right]$$

- For $A > A^*$ BH forms
  For $A < A^*$ matter disperses
- Near criticality geometrical quantities scale as[1]:
  $\ln(M) = \gamma \ln(A - A^*) + f(A - A^*)$
  f is periodic
- Both γ and period depend on n

[1] Choptuik, Phys. Rev. Lett. 70, 9 (1993)

Introduction

Gravitational collapse in GR and other theories

Higher dimensions interesting for several reasons:

- Asymptotic limit of critical exponent
- AdS/CFT correspondence
- Other higher dimensional theories

Spherical symmetry good starting point

Problems in higher dimensions

- Stability near $r = 0$
- Horizon radii decrease
- Time to formation lengthens

Define $\Phi = \psi_{,r}$

$$
\begin{aligned}
(\Pi_\psi)_{,t} =& \frac{1}{r^{n-2}}\left[r^{n-2}N\left(\Phi + \left(\frac{N_r}{N}\right)\Pi_\psi\right)\right]_{,r} \\
=& (n-2)\frac{1}{r}\left[N\left(\Phi + \left(\frac{N_r}{N}\right)\Pi_\psi\right)\right] + \left[N\left(\Phi + \left(\frac{N_r}{N}\right)\Pi_\psi\right)\right]_{,r}
\end{aligned}
$$

Unstable near $r = 0$
Use l'Hôpital's trick:[2]

$$\frac{f(r)}{r} = f_{,r} - r\left(\frac{f}{r}\right)_{,r}$$

[2] Maliborski, & Rostworowski. International Journal of Modern Physics A, 28(22n23), 1340020 (2013)

Boundary value at $r = 0$

$$\begin{aligned}\psi(r \approx 0, t) =& \psi_0(t) + \psi_2(t) r^2 + \psi_4(t) r^4 + \cdots \\ \Phi(r \approx 0, t) =& \Phi_0(t) r + \Phi_2(t) r^3 + \Phi_4(t) r^5 + \cdots\end{aligned}$$

Extrapolate for ψ and Π_ψ
$\Phi(r = 0, t) = 0$
In FDA need to extrapolate

$$\frac{1}{r}\left[N\left(\Phi + \left(\frac{N_r}{N}\right)\Pi_\psi\right)\right]$$

Methods for Higher Dimensions - AMR

Challenges:

1. Resolve large and small length scales
2. Run longer in higher dimensions

Use adaptive mesh refinement to overcome:

- Cover large computational domain
- Increase resolution only where necessary
- Resolve small-scale features efficiently
- 8 refinements by factor 4 $\Rightarrow \approx 6 \times 10^4$

Methods for Higher Dimensions - AMR

Accuracy and stability a challenge

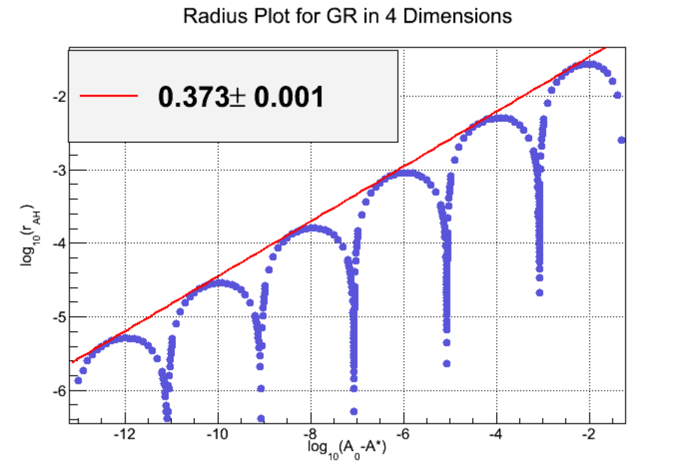
- Runge-Kutta 4 time integration
- 4th order spatial derivatives
- Dissipation (even near $r = 0$)[3]
- Higher $D \Rightarrow$ smaller Courant factor
- Higher $D \Rightarrow$ more dissipation

Results in longer simulations
Choice of initial data:

$$\psi(r, t = 0) = Ar^2 \exp\left[-\left(\frac{r - r_0}{B}\right)^2\right]$$

[3] ≈ 48 pts/wavelength, $\tau = 10^{-5}$

Preliminary Results - 4D

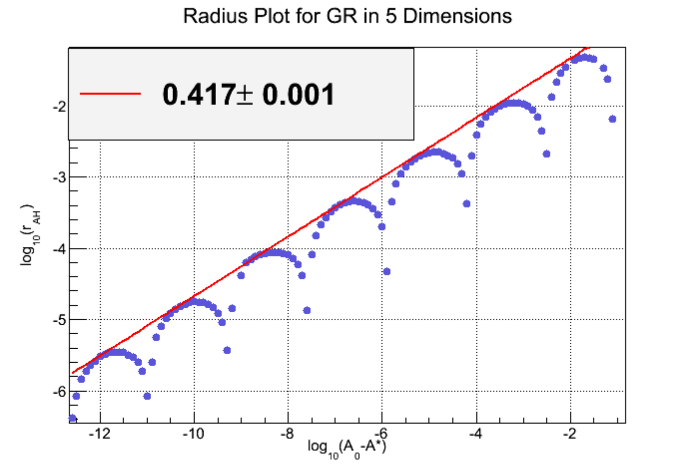


$\gamma = 0.373 \pm 0.001$, $\Delta = 3.45 \pm 0.03$ - Cusps for Δ

Agree with accepted values[4]

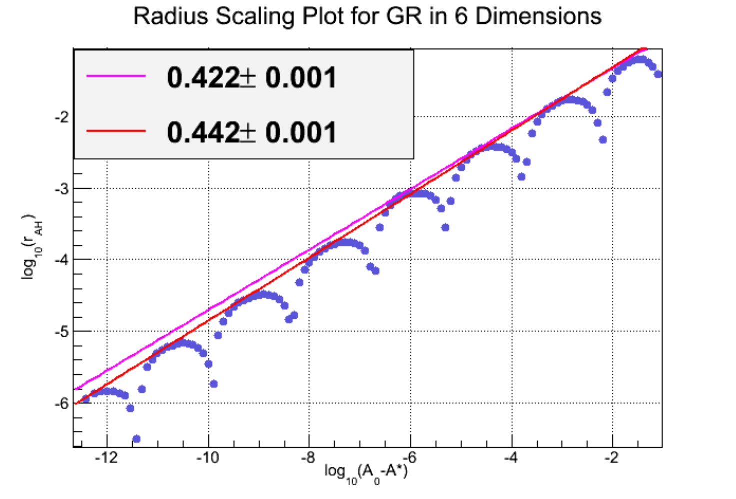
[4] Gundlach (1997) PhysRevD.55.695, Hamad & Stewart (1996) Class. Quantum Grav. 13 497

Preliminary Results - 5D



0.408 ± 0.008	Sorkin & Oren, Phys. Rev. D 71, 124005 (2005)
0.412 ± 0.004	Bland et al., Classical Quantum Gravity 22, 5355 (2005)
0.413 ± 0.002	Taves & Kunstatter(2011). PhysRevD.84.044034
0.416 ± 0.002	Taves & Kunstatter(2011). PhysRevD.84.044034

Preliminary Results - 6D



0.422 ± 0.008	Sorkin & Oren, Phys. Rev. D 71, 124005 (2005)
0.430 ± 0.003	Bland et al., Classical Quantum Gravity 22, 5355 (2005)
0.424	Garfinkle, Cutler, & Duncan, Phys. Rev. D 60, 104007 (1999)
0.429 ± 0.003	Taves & Kunstatter(2011). PhysRevD.84.044034
0.428 ± 0.002	Taves & Kunstatter(2011). PhysRevD.84.044034

Preliminary Results - 6D

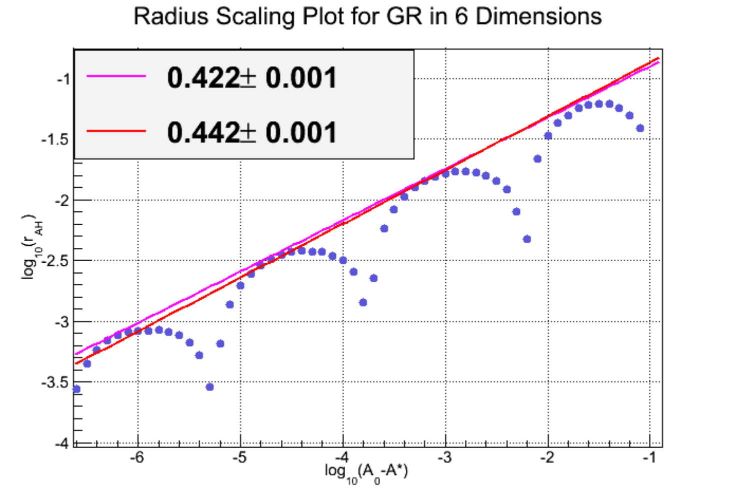


Figure: Zoom on Upper Portion 6D Scaling Plot

Preliminary Results - 6D

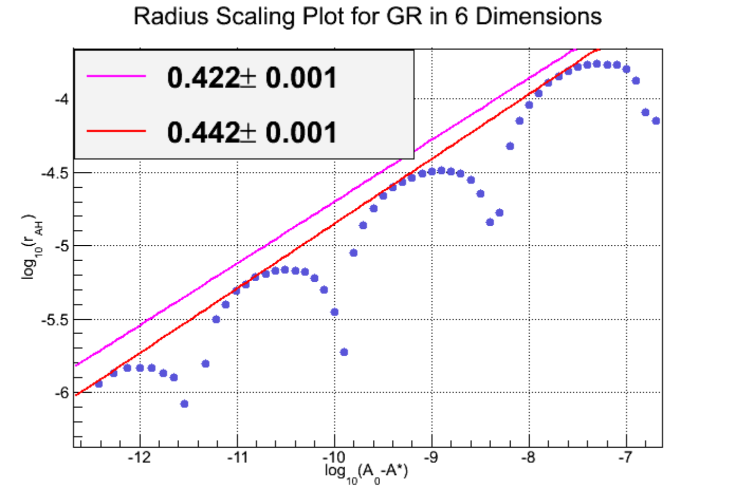


Figure: Zoom on Lower Portion 6D Scaling Plot

Summary

Investigating critical phenomena in higher D poses challenges:

- Stable equations - l'Hôpital's Trick
- Sufficient resolution - Adaptive Mesh Refinement
- Ensuring in critical region - Initial Data

Without care, results differ greatly - 6D example

Acknowledgments

Thanks to:

- The organizers of the conference
- My supervisor, Dr. Gabor Kunstatter
- NSERC
- The University of Winnipeg
- You, for listening