Examining AdS Spacetime in Einstein-Gauss-Bonnet Gravity

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Motivation

- AdS-CFT correspondence
- To examine gravitational collapse in higher dimensional AdS spacetime
- Wish to extend to Gauss-Bonnet
- Is AdS spacetime stable against the formation of black holes?
  - discrepancies between different groups

Motivation

- To repeat previous work done by Buchel et al., Bizoń et al., and Garfinkle but in different coordinates
- Motivated by results of the Gauss-Bonnet work done by others
- To be able to simulate many bounces off of infinity before finally collapsing to form a black hole

Background

- AdS-CFT correspondence
  - Relation between n-dimensional string theory and Conformal field theory on (n-1)-dimensional boundary surface
  - Difficulties in one case are simple in the other
  - CFT= scale invariant QFT (i.e. $x^\mu \to \lambda x^\mu$)

Background

- What is AdS spacetime?
  - Maximally symmetric solution to the Einstein equations
  - Negative cosmological constant
  - Massless scalar field can travel to infinity and back in finite time
  - Unlike Minkowski and de Sitter space, is unstable

Background: Conformal Diagrams

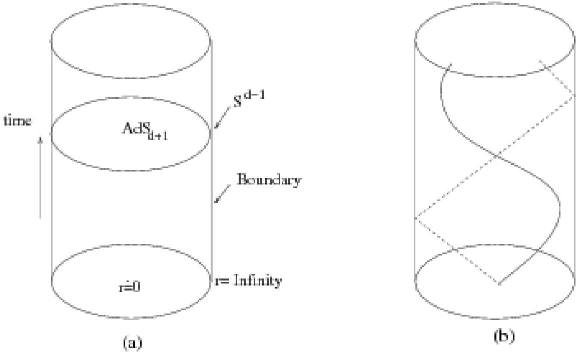

Figure: Maldacena, arXiv:1106.6073

Horizon radius vs amplitude for initial data

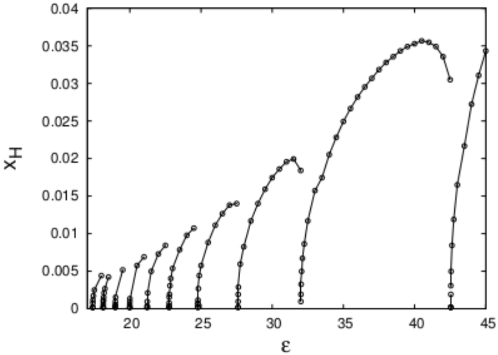

Figure: Bizoń, Rostworowski, arXiv:1104.3702

Metric:

$$ds^2 = -N^2 dt^2 + \Lambda^2 (dx + N_r dt)^2 + R^2 d\Omega^{(n-2)}$$

Gauge fixing conditions:

$$R = f(x) = \frac{Lx}{L - x} \tag{1}$$

$$\Lambda^{-2} = \frac{1}{f_{,x}^2} \left(1 - \frac{2\lambda f^2(x)}{(n-1)(n-2)} \right) \tag{2}$$

Equations of motion:

$$\psi_{,t} = \frac{\delta H}{\delta P_\psi} = N\left[\frac{\tilde{P}_\psi}{\Lambda} + \left(\frac{N_r}{N}\right)\psi_{,x}\right]$$

$$\tilde{P}_{(\psi),t} = \frac{1}{f(x)^{n-2}}\frac{\delta H}{\delta\psi} = \frac{1}{f(x)^{n-2}}\left[\frac{N}{\Lambda}f(x)^{n-2}\psi_{,x} + \left(\frac{N_r}{N}\right)\tilde{P}_\psi\right]_{,x}$$

Constraint Equation:

$$M_{,x} = f(x)^{n-2}\frac{f(x)_{,x}}{\Lambda}\left[\frac{1}{\Lambda}\left(\frac{\tilde{P}_\psi^2 + \psi_{,x}^2}{2}\right) + \left(\frac{N_r}{N}\right)\tilde{P}_\psi\psi_{,x}\right]$$

Consistency Condition:

$$\frac{N_{,x}}{N} = -\left[\frac{\Lambda}{f(x)_{,x}}\right]_{,x}\frac{f(x)_{,x}}{\Lambda} + \frac{f(x)^{n-2}\tilde{P}_\psi\psi_{,x}}{M_{,\left(\frac{N_r}{N}\right)}}\frac{f(x)_{,x}}{\Lambda}$$

Boundary Conditions - At the Origin

- At $R = 0$, set $N = 1$, $N_r = 0$
  - Setting $N = 1$ makes the proper time the coordinate time
  - $M = 0$
- Since $f(x)$ is a radial coordinate, thus ψ and P_ψ are regular. ie $\psi'(f(x) = 0, t) = 0$
- we define: $\tilde{P}_\psi = \frac{P_\psi}{f^{n-2}}$ and $P_\psi = \frac{f^{n-2}}{N}\left(\dot{\psi} - N_r\psi'\right)$

$$\psi(0,t) \approx \psi_0 + \psi_{(2)} f^2(x) + \psi_{(4)} f^4(x)$$
$$\tilde{P}_\psi(0,t) \approx P_{\psi,(0)} + P_{\psi,(2)} f^2(x) + P_{\psi,(4)} f^4(x)$$

Boundary Conditions - At the Origin: Ignoring Self Interaction

Ignoring self-interaction (i.e. $N_r \to 0$ and $N \to 1$) we obtain the spherically symmetric wave equation

$$\partial_t^2 \psi = (n-2)\frac{\partial_x f(x)}{f(x)}\partial_x \psi + \partial_x^2 \psi$$

Assume $\psi(x,t) = e^{i\omega t}\varphi(x)$:

$$0 = \omega^2 \varphi + (n-2)\frac{L}{x(L-x)}\partial_x \varphi + \partial_x^2 \varphi$$

Which gives the asymptotic solution:

$$\varphi(x) = \varphi_{(0)} + \varphi_{(2)}(\omega x)^2 + \varphi_{(4)}(\omega x)^4 + \varphi_{(6)}(\omega x)^6 + \cdots$$

Boundary Conditions - At Infinity

- At infinity the metric must tend to the Schwarzchild-AdS metric. Thus we construct a Taylor Series expansion about infinity
- We have chosen for our first gauge choice, $f(x) = \frac{Lx}{L-x}$
- Much more subtle than at $R = 0$

Boundary Conditions - At Infinity

At infinity, define $z = L - x$ so that as $x \to L, z \to 0$ and $x \to 0, z \to L$

Will need to solve Bessel's differential equation about $x = L$

$$\Phi(t, x \approx L) = (L-x)^{n-2}\left[\Phi_{(0)} + \Phi_{(2)}(L-x)^2 + \Phi_{(4)}(L-x)^4 + \cdots\right]$$

$$\Pi_\psi(t, x \approx L) = (L-x)^{n-1}\left[(\Pi_\psi)_0 + (\Pi_\psi)_2(L-x)^2 + (\Pi_\psi)_4(L-x)^4 + \cdots\right]$$

Current State and Future Goals

- Last summer: put our equations into the code to test them $\rightarrow$ issues
- Removed the gravitational self-interaction (i.e. set $N_r = 0$) $\rightarrow$ still issues
- Now have properly finite differenced equations and fully functional code

Thanks

- The conference organizers
- Dr. Gabor Kunstatter and Dr. Andrew Frey
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- University of Winnipeg
- You for listening!