In Search of de Sitter Solutions in String Theory

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based on arXiv:1110.0545 [hep-th]

with Stephen Green, Emil Martinec and Savdeep Sethi

and work in progress

Motivation

- Cosmological constant problem reminds us how poorly we really understand quantum gravity
- Unlike AdS and Minkowski spacetimes, quantum gravity in de Sitter remains truly mysterious
- Difficult to construct explicit dS solutions in string theory
- Difficulty arises from No-Go Theorem:
  - SUGRA satisfies SEC
    $$R_{00} \geq 0$$
  - de Sitter must violate SEC
- Our goal: Understand how string theory can evade No-Go

Outline

No-Go Theorem

- “No de Sitter compactifications of $D = 10/11$ supergravity"

Extremely simple and elegant (Gibbons ’84)

(See also de Wit et al., Maldecena-Nunez)

- Holds for all supergravity theories in $D = 10$ and 11
- Key point: SEC is satisfied in $D = 10/11$
- For concreteness, look at $D = 10$ $\mathcal{N} = 1$ sugra

SEC in $D = 10$ $\mathcal{N} = 1$ supergravity

$$S = \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}\, e^{-2\phi}\left[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4}\mathrm{tr}\,|F|^2\right],$$

In Einstein frame ($g \to e^{\phi/2} g$):

$$\begin{aligned} R_{MN} &= \frac{1}{2}\nabla_M\phi\nabla_N\phi + \frac{1}{4}e^{-\phi}H_{MPQ}H_N{}^{PQ} - \frac{1}{8}e^{-\phi}g_{MN}|H|^2 \\ &\quad + \frac{\alpha'}{4}e^{-\phi/2}\left[\mathrm{tr}\,F_{MP}F_N{}^P - \frac{1}{8}g_{MN}\mathrm{tr}\,|F|^2\right] \end{aligned}$$

$$-\nabla^2\phi = \frac{1}{2}e^{-\phi}|H|^2 + \frac{\alpha'}{8}e^{-\phi/2}\mathrm{tr}\,|F|^2$$

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In particular,

$$\begin{aligned} R_{00} \quad &= \quad \frac{1}{2}(\dot{\phi})^2 + \frac{1}{4}e^{-\phi}\left(H_{0IJ}H_0{}^{IJ} + \frac{1}{12}H_{IJK}H^{IJK}\right) \\ &\quad + \frac{\alpha'}{32}e^{-\phi/2}\left(7F_{0I}F_0{}^I + \frac{1}{2}F_{IJ}F^{IJ}\right) \geq 0 \end{aligned}$$

Proof of the No-Go Theorem

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$$R_{00}^{(D)} \geq 0$$

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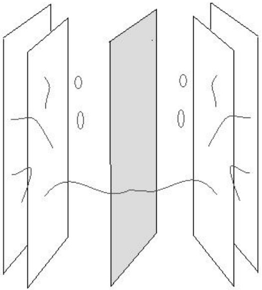
4. Multiply by W^{D-2}, and integrate over Y.

As long as $W(y) \neq 0$, guaranteed that $R_{00}^{(d)} \geq 0$.

SEC and String Theory

Rule out inflation/dS in string theory? No

- String theory is not just supergravity
- Have objects like O-planes, with negative tension



- These can violate SEC and support higher derivative interactions

SEC and String Theory

- O-planes are difficult to work with:
    - delta function sources
    - W(y) singular
    - SUGRA breaks down
- String dualities map O-planes to *smooth* geometric configurations in heterotic string theory
- O-planes effects completely captured by R^2 corrections to sugra

Heterotic Supergravity

$D = 10$ $\mathcal{N} = 1$ supergravity with specific $\alpha' \sim 1/M_{Pl}^2$ corrections, and field definitions

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g}\, e^{-2\phi} \Big[R + 4(\partial\phi)^2 - \frac{1}{2}|H|^2 - \frac{\alpha'}{4} \left(\mathrm{tr}\,|F|^2 - \mathrm{tr}\,|R_+|^2 \right) + O(\alpha'^3) \Big],$$

where

$$\mathrm{tr}\,|R_+|^2 = \frac{1}{2} R_{MNAB}(\Omega_+) R^{MNAB}(\Omega_+)$$

$$\Omega_{\pm}^{AB}{}_M = \Omega^{AB}{}_M \pm \frac{1}{2} H^{AB}{}_M + O(\alpha')$$

$$H = dB + \frac{\alpha'}{4} \left[\mathrm{CS}(\Omega_+) - \mathrm{CS}(A) \right],$$

Modified Equations of Motion

In Einstein frame,

- Einstein equation (trace-reversed)

$$R_{MN} = \ldots + \frac{\alpha'}{4} e^{-\phi/2} \left[\frac{1}{8} g_{MN} \mathrm{tr} \left| R_+ \right|^2 - R_{+\,MPAB} R_{+\,N}{}^{PAB} \right] + O(\alpha'^3)$$

- In particular,

$$\begin{aligned} R_{00} \quad &= \quad \ldots + \frac{\alpha'}{16} e^{-\phi/2} \left[\left(7 R_{+\,0I0J} R_{+\,0}{}^{I}{}_{0}{}^{J} + \frac{1}{2} R_{+\,IJ0K} R_{+}{}^{IJ}{}_{0}{}^{K} \right) \right. \\ &\qquad \left. - \frac{1}{2} \left(7 R_{+\,0IJK} R_{+\,0}{}^{IJK} + \frac{1}{2} R_{+\,IJKL} R_{+}{}^{IJKL} \right) \right] \end{aligned}$$

- No longer positive definite

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- This will ultimately forbid SEC violations

Caveats

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  - One string-loop comes in at $O(\alpha'^3)$
- Ignore non-perturbative g_s corrections
  - $NS5$-branes are just point-like instantons (no SEC violation)
  - Other spacetime non-perturbative effects, like gaugino condensation, could alter this picture

Ansatz

- We'll take an extremely simple ansatz for our spacetime fields:

$$
\begin{aligned}
ds^2 &= e^{2\omega(y)}\left(\hat{g}_{\mu\nu}(x)dx^\mu dx^\nu + \hat{g}_{mn}(y)dy^m dy^n\right) \\
H &= H_{mnp}(y)dy^m dy^n dy^p \\
F &= F_{mn}(y)dy^m dy^n \\
\phi &= \phi(y)
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- In particular $\dot{\phi} = 0$.
- Ignore axion h, $dh = *H$.
- Ignore all other light scalars from compactifying.

$\Rightarrow$ No dynamical scalars (moduli or otherwise).

Comment on different frames

- Action written with string frame metric, g
- EoM/SEC best studied in Einstein frame,

$$g = e^{\phi/2}\tilde{g}$$

- In practice, useful to use product frame metric

$$g = e^{\phi/2}W^2\hat{g}$$

- Define

$$\omega = \log W + \frac{1}{4}\phi$$

conformal factor between string and product frame

Method

- We don't know form of internal fields $R_{mnpq}(\Omega_+)$, H_{mnp}, etc.
- Use the dilaton equation

$$\tilde{\nabla}^M \tilde{\nabla}_M \phi + \frac{1}{2} e^{-\phi} |H|^2 + \frac{\alpha'}{8} e^{-\phi/2} \left(\mathrm{tr}\, |F|^2 - \mathrm{tr}\, |R_+|^2 \right) = O(\alpha'^2)$$

to eliminate these from the Einstein equations

$$\begin{aligned} \tilde{R}_{MN} &= \frac{1}{4} \tilde{g}_{MN} \tilde{\nabla}^P \tilde{\nabla}_P \phi + \frac{1}{2} \tilde{\nabla}_M \phi \tilde{\nabla}_N \phi + \frac{1}{4} e^{-\phi} H_{MPQ} H_N{}^{PQ} \\ &+ \frac{\alpha'}{4} e^{-\phi/2} \left[\mathrm{tr}\, F_{MP} F_N{}^P - R_{+\,MPAB} R_{+\,N}{}^{PAB} \right] . \end{aligned}$$

Method

Write in product frame, look at spacetime components:

$$\begin{aligned}\hat{R}_{\mu\nu} &= \hat{g}_{\mu\nu} W^{-8} \hat{g}^{mn} \hat{\nabla}_m \left(W^8 \hat{\nabla}_n \omega \right) \\ &-\frac{\alpha'}{4} e^{-2\omega} \left[\hat{R}_{\mu\rho\lambda}{}^{\sigma} \hat{R}_{\nu}{}^{\rho\lambda}{}_{\sigma} - 4 \hat{R}_{\mu\nu} |\hat{\nabla}_m \omega|^2 \right. \\ &\left. + 2\hat{g}_{\mu\nu} \left(3 \left(|\hat{\nabla}_m \omega|^2 \right)^2 + 2|X_{mn}|^2 + \frac{1}{2} e^{-4\omega} |H_{mn}{}^{p} \hat{\nabla}_p \omega|^2 \right) \right]\end{aligned}$$

where

$$X_{mn}(\omega) = \hat{\nabla}_m \omega \hat{\nabla}_n \omega - \hat{\nabla}_m \hat{\nabla}_n \omega - \hat{g}_{mn} |\hat{\nabla}_p \omega|^2$$

Maximally symmetric spacetimes

- In the case where $\hat{R}_{\mu\nu} = \Lambda \hat{g}_{\mu\nu}$, this reduces to

$$W^{-8}\hat{\nabla}^m \left(W^8 \hat{\nabla}_m \omega \right) = \Lambda$$
$$+\frac{\alpha'}{2} e^{-2\omega} \left[\frac{1}{3} \left(\Lambda - 3|\hat{\nabla}_m \omega|^2 \right)^2 + 2|X_{mn}|^2 + \frac{1}{2} e^{-4\omega} |H_{mn}{}^p \hat{\nabla}_p \omega|^2 \right]$$

- Note: R^2 terms and $|\hat{\nabla}\omega|^2$ combine into perfect square!
- As in no-go thm, multiply by W^8 and integrate over Y

Result:

$$\Lambda = -\frac{\alpha'}{2V'}\int_{\mathcal{M}} d^6y\sqrt{\hat{g}}W^8 e^{-2\omega}\Big[3|\hat{\nabla}_m\omega|^2 + 2|X_{mn}|^2 + \frac{1}{2}e^{-4\omega}|H_{mn}{}^p\hat{\nabla}_p\omega|^2\Big] + O(\alpha'^2)$$

where $V' = \int_{\mathcal{M}} d^6y\sqrt{\hat{g}}W^8$,
and $X_{mn}(\omega) = \hat{\nabla}_m\omega\hat{\nabla}_n\omega - \hat{\nabla}_m\hat{\nabla}_n\omega - \hat{g}_{mn}|\hat{\nabla}_p\omega|^2$

- Already see that $\Lambda \leq 0$
- No de Sitter solutions in this setup
- In fact $\Lambda < 0$ also forbidden at this level

Result:

As pointed out in arXiv:1204.0807 (Gautason, Junghans, Zagermann):

$$\nabla\omega \sim O(\alpha')$$

- Dilaton EoM:

$$-\tilde{\nabla}^2\phi = \frac{1}{2}e^{-\phi}|H|^2 + O(\alpha') \quad \Rightarrow \quad W^{10}e^{-\phi}|H|^2 \sim \tilde{\nabla}^2\phi \sim O(\alpha')$$

- $4d$ trace of Einstein equation:

$$W^{10}\tilde{R}_4 = W^8\hat{R}_4 - \frac{9}{2}\hat{\nabla}^2 W^8 = -\frac{1}{2}W^{10}e^{-\phi}|H|^2 + \mathcal{O}(\alpha')$$

- Put these together:

$$\nabla\omega \sim O(\alpha') \quad \Rightarrow \quad \Lambda \sim O(\alpha'^2)$$

- Can extend to all orders in α', at string tree level ($S \propto e^{-2\phi}$)

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How to go further? Look back at assumptions:

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  - gaugino condensation

Gaugino Condensation

- Ignored fermions, since *typically* vanish in vacuum
- In IR limit of $d = 4$ $\mathcal{N} = 1$ SYM, gauginos χ form scalar condensate:

$$\langle \bar{\chi}\chi \rangle \sim \Lambda_{YM}^3 \sim \mu^3 e^{-1/g_{YM}^2}$$

- Embedded into string theory, gauge and string couplings related

$$g_{YM} \sim g_s = e^{\phi}$$

- Non-perturbative in g_s

$D=10$ $\mathcal{N}=1$ Supergravity with Gauginos

$$
\begin{aligned}
S \quad &= \quad \frac{1}{2\kappa^2}\int d^{10}x\sqrt{-g}e^{-2\phi}\Big[R+4(\partial\phi)^2-\frac{1}{2}|T|^2 \\
&\qquad -\frac{\alpha'}{4}\left(\mathrm{tr}\,|F|^2-\mathrm{tr}\,|R_+|^2+2\mathrm{tr}\,\bar{\chi}\not{D}\chi\right)+O(\alpha'^3)\Big]
\end{aligned}
$$

where

$$
T_{MNP}=H_{MNP}+\frac{\alpha'}{8}\mathrm{tr}\,\bar{\chi}\Gamma_{MNP}\chi
$$

$$
\begin{aligned}
R_{00} \quad &= \quad \frac{1}{2}(\dot{\phi})^2+\frac{1}{4}e^{-\phi}\left(T_{0IJ}T_0{}^{IJ}+\frac{1}{12}T_{IJK}T^{IJK}\right) \\
&\qquad +\frac{\alpha'}{32}e^{-\phi/2}\left(7F_{0I}F_0{}^I+\frac{1}{2}F_{IJ}F^{IJ}-(R_+^2)_{00}\right)
\end{aligned}
$$

Ignoring R_+^2 terms, satisfies SEC

- Repeat same analysis:

$$(tot.\ der.) \;=\; \Lambda + \frac{\alpha'}{64} e^{-2\omega} \mathrm{tr}\,(\bar{\chi}\Gamma^{mnp}\chi) H_{mnp} + \frac{1}{8}\left(\frac{\alpha'}{8}\mathrm{tr}\,\bar{\chi}\Gamma^{mnp}\chi\right)^2 + \alpha' \sum |f(\nabla\omega)|^2 + O(\alpha'^3)$$

- As before, can easily show $\nabla\omega \sim O(\alpha')$
- If not for term linear in α', would have $\Lambda \leq 0$:

$$\Lambda \sim -(\alpha' \Lambda_{YM}^3)^2$$

consistent with explicit AdS constructions,
eg. hep-th:0507202 (Frey,Lippert)

Conclusion

Summary

- Looked for SEC violation from $O(\alpha')$ effects in heterotic supergravity
- these effects are dual to (neg. tension) O-planes in other duality frames
- Dilaton plays important role in constraining size of SEC violation

Results

- de Sitter not possible by perturbative α' or (known) non-perturbative g_s corrections
- Non-perturbative α' or perturbative g_s still possibility, but seems unlikely

Take Away:

Most likely, de Sitter compactifications not compatible with α' expansion