Kerr-AdS black holes and force-free magnetospheres

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Based on
XW and A. Ritz, arXiv:1402.1452 (PRD, to appear)

Outline

1. Overview/Motivation
2. Discuss aspects of Blandford-Znajek (BZ) process (force-free magnetosphere as key ingredient)
3. Solution for BZ in AdS space
4. Dual field theory interpretation under AdS/CFT

Overview/Motivation

Rotating (Kerr) BHs can lose energy via, e.g., Penrose, superradiance or BZ process.

Interested in generalizing BZ to Kerr-AdS (as has been done for e.g. superradiance)

Main question: What is the dual description of force-free magnetosphere?

BZ mechanism

[R. D. Blandford and R. L. Znajek, MNRAS 179, 433 (1977)]

Intuitive picture: Rotating black holes immersed in external magnetic field $\rightarrow$ Poynting flux (power jets: AGN etc.)

Modeling: force-free magnetosphere filled with plasma of negligible inertia $\rightarrow$ conservation equation & degenerate EM field (simple bi-vector):

$$(T_{\mathrm{EM}})_{\mu\nu}^{;\nu} = F_{\nu\mu} J^{\nu} = 0 \quad \Rightarrow \quad F \wedge F = 0,\ F = \eta \wedge \xi.$$

BZ's (split) monopole ansatz: spin up a test monopole field and solve in slow rotation limit to $\mathcal{O}(a^2)$, which we generalize to AdS.

BZ & membrane paradigm

Another connection of BZ & AdS/CFT is through membrane paradigm, which was partially motivated by BZ.

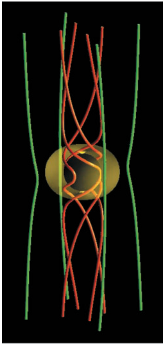
AdS/CFT is the modern version of membrane paradigm, both being holographic principles.
(There are studies on the connections of the two under fluid/gravity correspondence.)

For BZ, the membrane ('stretched horizon') endowed with e.g., conductivity, serves as the unipolar inductor.

BZ & ergosphere

Nowadays it is accepted that the role of 'unipolar inductor' is played by ergosphere, present in rotating geometries ($\beta^i \neq 0$).

Plasma are dragged into negative energy orbits and twists of field line propagates away in form of Poynting flux.



How is the appearance of an ergosphere identified in field theory? (No conclusive answer yet.)

BZ in Kerr-AdS

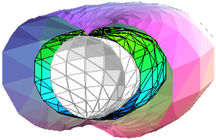
Current approach: solve for $A_{t(\varphi)}$ in BZ and interpret the falloff using AdS/CFT dictionary, expecting field theory with chemical potential and charge density.

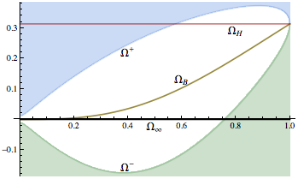
Kerr-AdS metric omitted but the following expressions will be used:
$\Delta_r = (r^2 + a^2)(1 + \frac{r^2}{l^2}) - 2mr, \Sigma = r^2 + a^2\cos^2\theta, \Delta_\theta = 1 - \frac{a^2}{l^2}\cos^2\theta, \Xi = 1 - \frac{a^2}{l^2}.$

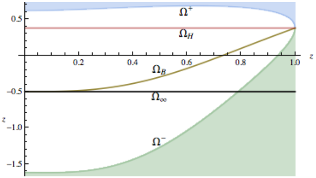
Non-unique energy-defining Killing vector:

$$K_\Omega = \xi_{(t)} + \Omega\xi_{(\varphi)}, \quad \Omega = -\frac{a}{l^2}(= \Omega_\infty),\, 0,\, \Omega_H, \ldots$$

Spacelike $K_\Omega \Rightarrow$ ingoing "energy" flux on horizon for generic $T^{\mu\nu}$.





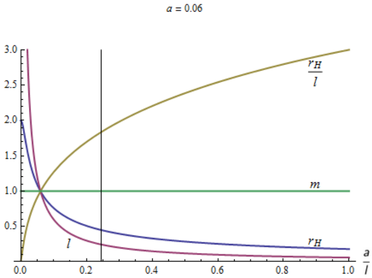


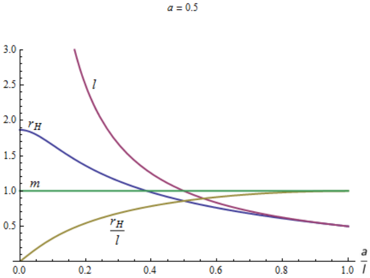
BZ in Kerr-AdS: slowly rotating monopole

Compared to Kerr, one additional parameter l. Nevertheless, $\frac{a}{m} = \epsilon \ll 1$ is still a good criterion for "far from extremality": $\frac{m_{\text{extr}}}{m} \sim \epsilon$, since $\frac{a}{m_{\text{extr}}} \sim \mathcal{O}(1)$.

In practice, treat 'a' small relative to other scales. Subcases characterized by BH size $\frac{r_H}{l}$: (large BH implies slow rotation.)

1. $a \ll m \sim \frac{r_H}{2} \ll l$ (small BH, Kerr limit)
2. $a \ll m \sim r_H \sim l$ (transition from small to large BHs)
3. $a \ll l \ll r_H \ll m$, e.g., $\epsilon^{\frac{1}{2}} \cdot \epsilon^{\frac{1}{4}} \cdot \epsilon^{\frac{1}{4}}$ (large BHs)





BZ in Kerr-AdS: setups

Up to $\mathcal{O}(a^2)$ and using unit $m = 1$, boils down to only one free parameter $r_1 = r_H(a = 0)$:

$$r_H \approx r_1, \quad \Omega_H \approx \frac{a}{r_1^2}, \quad l^2 = \frac{r_1^3}{2 - r_1}, \quad [0 < r_1 < 2 \text{ (Kerr limit)}]$$

Degenerate $F_{\mu\nu}$ described by (assuming stationarity, axisymmetry)

$$\omega(A_\varphi) \equiv -\frac{A_{t,\theta}}{A_{\varphi,\theta}} = -\frac{A_{t,r}}{A_{\varphi,r}}, \quad \text{(angular frequency of magnetic field lines)}$$

$$B_T(A_\varphi) = (g_{\varphi\varphi} g_{tt} - g_{\varphi t}^2) B^\varphi. \quad \text{(toroidal magnetic field)}$$

Small 'a' expansion,

$$A_\varphi = A_\varphi^{(0)} + a^2 A_\varphi^{(2)}, \quad \omega = a\omega^{(1)}, \quad B_T = aB_T^{(1)}$$

BZ in Kerr-AdS: split monopole solution

$$A_\varphi^{(0)} = -C\cos\theta, \quad A_\varphi^{(2)} = f(r)\cos\theta\sin^2\theta$$
$$\omega^{(1)}(\theta) = \omega^{(1)}, \quad B_T^{(1)}(\theta) = B_T^c \sin^2\theta$$

Conservation equation $T_{\mu\nu}^{;\nu} = 0$ becomes

$$f''(z) + \frac{2z(3z - r_1)}{(z-1)[2z^2 - (r_1 - 2)(z+1)]} f'(z) + \frac{6 r_1 f(z)}{(z-1)[2z^2 - (r_1 - 2)(z+1)]} - C\frac{4(z^2 - 2\omega^{(1)} r_1^2)(z^2 + z + 1) + 2r_1(z+1)}{(z-1)[2z^2 - (r_1-2)(z+1)]^2 r_1} = 0, \quad (z \equiv \frac{r_1}{r})$$

- $B_T = C\sin^2\theta(\omega - a/r_1^2)$ by regularity on the horizon $z = 1$.
- Lack of singular point at $z = 0$ leaves ω arbitrary. (red terms)

BZ in Kerr-AdS: series solution

For Kerr, $f(z) \sim z + \mathcal{O}(z^2 \ln z), \quad \omega = \Omega_H/2$.
For Kerr-AdS, seek regular series solutions

$$f_0(z) = \sum_{n=1}^{\infty} c_n z^n, \qquad f_1(z) = \sum_{n=0}^{\infty} b_n (z-1)^n,$$

finding, $f_0 = f_0(z; \omega^{(1)}, c_1)$, $f_1 = f_1(z; \omega^{(1)}, b_0)$.

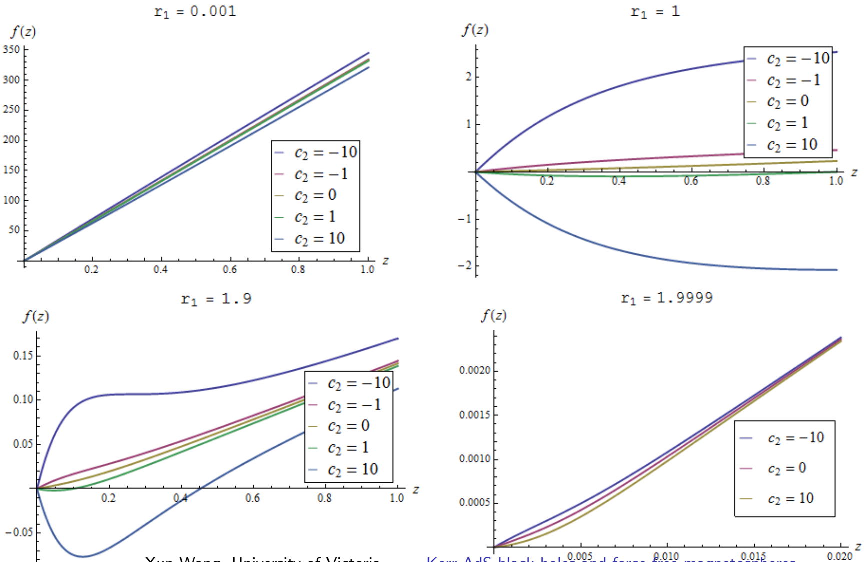
Matching $f_1(0) = 0$, $f_1'(0) = c_1$ fixes c_1, b_0 in terms of $\omega^{(1)}$.

In fact, $\omega^{(1)} \Leftrightarrow c_2$, i.e., concavity of $f(z)$. Explicitly,

$$\omega^{(1)} = \frac{1}{4r_1} + \frac{c_2}{C}\frac{(r_1-2)^2}{4r_1}.$$

BZ in Kerr-AdS: numeric solution ($\approx$ series solution)

Obtain a set of solutions labeled by $c_2 \Leftrightarrow \omega^{(1)}$, for each r_1:



BZ in Kerr-AdS: matching to rotating monopole in AdS

ω_{Kerr} fixed by matching onto Michel's rotating monopole solution in flat ST. (Giving a second relation between B_T & ω besides that from horizon regularity.)

Arbitrariness of ω persists even in pure AdS, for the same reason as in BH case: lack of singular point at infinity.

E.g., the perturbed ansatz (with *analytic* solution for $f(y)$)

$$A_\Phi = \text{`monopole'} + a^2 f(y)$$

only fixes $(B_T^c)^2 - \omega^2 = -\frac{c_2}{l^4}$, where c_2 is an overall factor of $f(y)$.

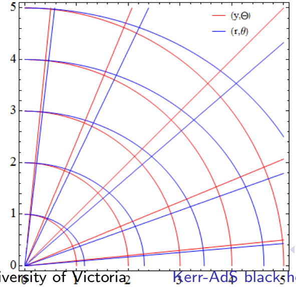
BZ in Kerr-AdS: matching to rotating monopole in AdS

An ambiguity even with an unperturbed monopole — there are two coordinate systems in which to define 'monopole':

1. in Boyer-Lindquist (r, θ), as $A_\varphi = -C\cos\theta$
2. in standard AdS (y, Θ), as $A_\Phi = -C\cos\Theta$.

At large radius, $y = r\sqrt{\frac{\Delta_\theta}{\Xi}}$, $\cos\Theta = \cos\theta\sqrt{\frac{\Xi}{\Delta_\theta}}$, but coincide when $a = 0$, or in Kerr limit.

When spinning up a BH carrying test monopole charge, no good way to tell which of 1 & 2 is the 'true' asymptotic configuration.



BZ in Kerr-AdS: dual field theory interpretation

It is instructive to make connection with 'large Kerr-Newman-AdS/charged fluid' correspondence.

For small magnetic charge C,

$$\begin{aligned} ds^2_{\text{KNAdS}} &= ds^2_{\text{KAdS}} + \mathcal{O}(C^2) \\ A_{\text{KNAdS}} &= -C\cos\theta(d\varphi - \frac{a}{r^2}\,dt) + \mathcal{O}(a^3) \\ A_{\text{BZ}} &= -C\cos\theta(d\varphi - a\omega^{(1)}\,dt) + \mathcal{O}(a^2) \end{aligned}$$

According to $A_t \sim \mu + \rho/r + \cdots$, $\rho = 0$ and in fact,

$$\mu I = A_\mu K^\mu_{\Omega_H}|_{r\to\infty} - A_\mu K^\mu_{\Omega_H}|_{r\to r_H} = 0 + \mathcal{O}(a^3).$$

To linear order in C & a, can adopt results from K(N)AdS/CFT. Magnetic field rotating in neutral fluid without constraints on ω.

BZ in Kerr-AdS: fluxes

$\delta E = T\delta S + (\Omega_H - \Omega)\delta L, \quad (\Omega \text{ as in } K_\Omega = \xi_{(t)} + \Omega \xi_{(\varphi)})$.
For BZ,

$$\delta E \propto T^r_t + \Omega T^r_\varphi = r^{-2}(\omega - \Omega_H)(\omega - \Omega)A^2_{\varphi,\theta},$$
$$\delta L \propto -T^r_\varphi = r^{-2}(\omega - \Omega_H)A^2_{\varphi,\theta},$$
$$T\delta S \propto T^r_t + \Omega_H T^r_\varphi = r^{-2}(\omega - \Omega_H)^2 A^2_{\varphi,\theta} \geq 0.$$

δE, $\delta L < 0$ if $\Omega < \omega < \Omega_H$.

- If $\Omega = \Omega_H$, possible for large BHs, simply $\delta E = T\delta S \geq 0$. No instability or spin down.
- If $\Omega = \Omega_\infty$, ergosphere and energy extraction are present. This is inferred by AdS/CFT, since angular velocity of (the matter in) boundary Einstein universe is measured w.r.t K_{Ω_∞}, given by $\Omega_H - \Omega_\infty$.
[Agrees with Hawking & Reall, PRD 61, 024014 (1999).]

BZ in Kerr-AdS: thermodynamics

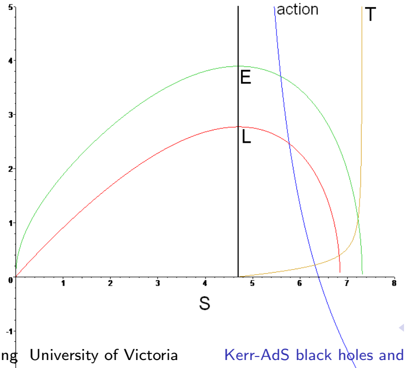
2nd choice also makes 1st law exact differentials.
Write T, Ω_H, Ω_∞ as functions of (S, L), and for BZ,

$$\frac{\mathrm{d}L(S)}{\mathrm{d}S} = \frac{T}{\omega - \Omega_H}, \qquad \frac{\mathrm{d}E(L(S), S)}{\mathrm{d}S} = T\frac{\omega - \Omega_\infty}{\omega - \Omega_H}.$$

For $\omega = \Omega_H/2$, find integral curves of 1st law in (E, S, L) space:

$$L(S) = S\sqrt{C - \frac{3S^2 + 8\pi S + 2\pi^2 \ln S}{4\pi^4}}, \quad E(S) = \sqrt{S(S+\pi)}\sqrt{C - \frac{3S^2 + 7\pi S + 2\pi^2 \ln S - \pi^2}{4\pi^2}}.$$

Non-extreme BHs lie right to the vertical line.



Thanks!